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OPTIMIZATION OF WALL THICKNESS FOR A CONSTANT HEAT FLUX INTERMITTENT OPERATION FURNACE

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NOMENCLATURE

- F, heat flux per unit area;
- h, heat transfer coefficient;
- k, thermal conductivity of wall material;
- l, wall thickness;
- L, dimensionless wall thickness (hl/k)—Biot modulus;
- t, time to reach specified inside wall temperature;
- t_∞, time for semi-infinite solid to reach specified inside wall temperature;
- T, dimensionless time (t/t_∞);
- x, distance from inside surface of wall.

Greek symbols

- α, thermal diffusivity;
- γ, defined by equation (4);
- θ, temperature referred to ambient;
- Θ, dimensionless temperature $(\theta h/F_i)$.

THE ELECTRIC furnace with plain walls having constant power input may be approximated by a one-dimensional constant wall heat flux model. For operation on an intermittent basis a careful consideration of wall thickness is required. It would appear on preliminary consideration that if the wall is too thin the required temperature may not be reached, and if it is too thick an unnecessary amount of heat may be absorbed by the wall, indicating the possibility of an optimum wall thickness for minimum warm up time.

Although data are readily available for the transient behaviour of plain walls with specified inner surface temperature and convective heat loss at the outside, the equivalent data for the plain wall with constant heat flux does not appear to be available in useable form. The analytical solution for the problem is given in Carslaw and Jaeger [1] but it is not in a form which enables a direct examination of the present problem with sufficient accuracy to be of use. A new

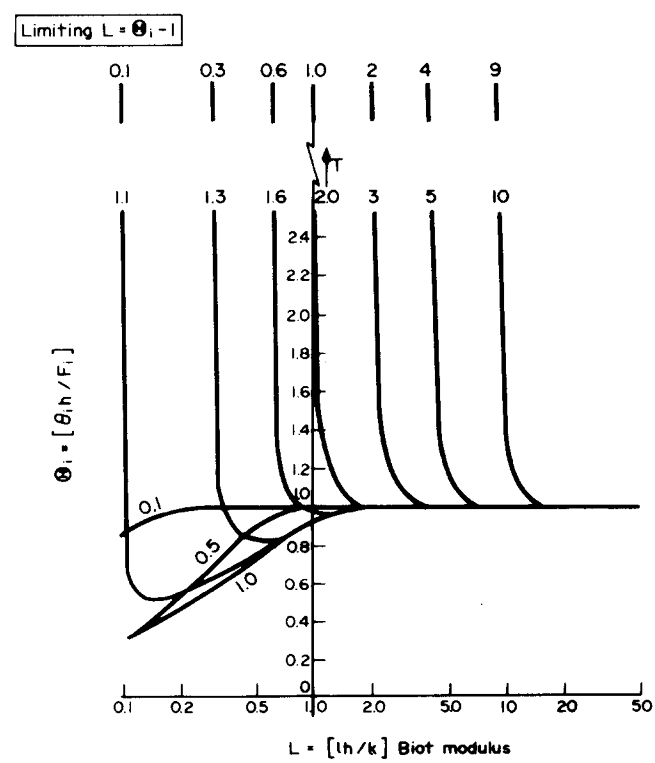

FIG. 1. Dimensionless time to reach a specified inner temperature function with varying wall thickness function.

computation of the result given below is made and the results presented in a format suited to the problem.

The one-dimensional equation for transient conduction through a plain wall with constant properties is

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\,\frac{\partial \theta}{\partial t}. \tag{1}$$

The boundary conditions are constant heat flux F_i at the inside and convective loss of heat at the outside

$$F_0 = -k \left.\frac{\partial \theta}{\partial x}\right|_0 = h\theta_0$$

and

$$\theta = 0 \quad \text{when } t = 0. \tag{2}$$

For a constant heat transfer coefficient the solution for the temperature field may be written as

$$\Theta = 1 + L(1 - x/l) - \sum_{n=1}^{\infty} \frac{2L(\gamma_n^2 + L^2)\cos\frac{\gamma_n x}{l}\exp - \frac{\gamma_n^2 \alpha t}{l^2}}{\gamma_n^2(L + L^2 + \gamma_n^2)}, \tag{3}$$

where γ_n, $n = 1, 2\ldots$ are the n positive roots of

$$\gamma \tan \gamma = L. \tag{4}$$

To normalize the time taken to reach a specified inside temperature parameter as wall thickness is raised, it is not convenient to use the conventional Fourier modulus, since that is a function of wall thickness. Instead the time t_∞ for the inside surface temperature of an infinitely thick wall to reach θ_i is used, where

$$t_\infty = \frac{\pi}{\alpha}\left(\frac{\theta_i k}{2F_i}\right)^2. \tag{5}$$

The variation of dimensionless time with wall thickness parameter for various values of inside temperature parameter is given in Fig. 1. It is seen that the result divides into two regimes separated by the condition that $\Theta_i = 1$. Below this value a totally non-equilibrium situation exists where the outside surface temperature will never be sufficient to dissipate the applied heat flux by convection. If $\Theta_i > 1$ a stable equilibrium is possible with the wall saturated and all the heat dissipated by convection ($\Theta_0 = 1$). The inside equilibrium temperature depends on the wall thickness: for a specified value of Θ_i there is a lower limit to the wall thickness given by

$$L_{\min} = \Theta_i - 1. \tag{6}$$

A minimum time to reach the inside wall temperature is exhibited if the wall thickness parameter is slightly greater than the limiting value. The minimum is however only evident for values of $\Theta_i < 2$.

For thick walls t is obviously asymptotic to the line for the semi-infinite solid.

A thickness chosen to minimise the heating time results in an outside wall temperature which may be close to the inside temperature, and this may become the controlling factor in design of the wall. The relationship between outside and inside wall temperature parameter is shown in Fig. 2.

A design specification will normally contain a maximum outside wall temperature. Thus, if the optimum wall obtained from Fig. 1 results in too high a surface temperature, it will be necessary to use Fig. 2 to obtain the minimum wall thickness, and to use Fig. 1 to read off the time.

The practical use of this data is limited to walls with large length/thickness ratios having small edge effects. The use of weighted average values for thermal conductivity and specific heat should prove satisfactory. The most difficult parameter is the heat transfer coefficient. For purely natural convection this is normally proportional to $\theta_0^{1/4}$. For low Reynolds number forced convection there is no simple temperature dependence but the influence of increasing mean boundary layer temperature on fluid properties is such as to reduce the value of h. In general, a combination of free and forced convection may well exist, but changes in conditions will alter the effect of outside wall temperature on h, which may vary by a significant amount. If high surface temperatures are reached further temperature dependence will be caused by radiation.

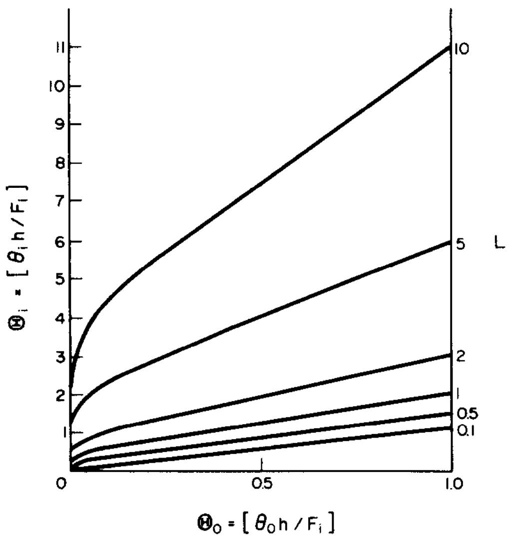

FIG. 2. Variation of outside with inside wall temperature function for varying thickness function.

CONCLUSION

Complete data is presented in Figs. 1 and 2 for the preliminary selection of plain furnace walls with constant heat flux in terms of four parameters Θ_i, Θ_0, L and T. Optimal wall thickness can be found for the range $1 < \Theta_i < 2$. For $\Theta_i < 1$, the outside temperature of the wall cannot achieve steady state. For $\Theta_i > 2$, an infinitely thick wall will achieve the prescribed inside temperature in the shortest time; but the time will not be appreciably longer for walls of finite thickness having L in the range one and a half to twice the value of Θ_i. It is possible, for a specified inside surface temperature parameter, to choose the design criterion either as the wall thickness giving the minimum heating time, or that giving a maximum outside wall temperature. In either case the auxiliary data are available from the alternative graph.

REFERENCE

1. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, p. 125. Clarendon Press, Oxford (1959).